Theory on level attraction

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Enhancement of electromagnetically induced transparency in metamaterials using long range coupling mediated by a hyperbolic material

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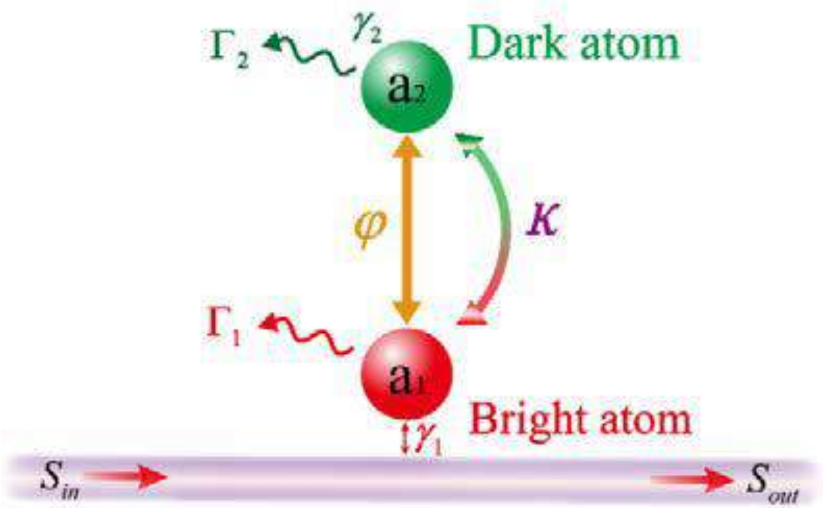


$$\tilde{S}_{in} = S_{in} \cdot e^{i\omega t}$$
$$\tilde{a}_1 = a_1 \cdot e^{i\omega t}$$
$$\tilde{a}_2 = a_2 \cdot e^{i\omega t}$$

Dynamic equations

Bright atom — Driving force

$$\frac{d\tilde{a}_1}{dt} = (i\omega_1 - \Gamma_1 - \gamma_1)\tilde{a}_1 + i\sqrt{\gamma_1}\tilde{S}_{in} + i\left(\kappa + i\sqrt{\gamma_1\gamma_2}e^{-i\phi}\right)\tilde{a}_2$$

$$\frac{d\tilde{a}_2}{dt} = (i\omega_2 - \Gamma_2 - \gamma_2)\tilde{a}_2 + i\left(\kappa + i\sqrt{\gamma_1\gamma_2}e^{-i\phi}\right)\tilde{a}_1$$

Dark atom — Near field coupling — Far field coupling

$\Gamma_{1,2}$ - dissipative loss / intrinsic damping (Origin from Ohmic loss [1])

$\gamma_{1,2}$ - radiative loss / extrinsic damping (Origin from structure [1])

$\phi = kl$ - phase difference from the separation l and wave number k

[1]. Sun, Yong, et al. "Experimental demonstration of a coherent perfect absorber with PT phase transition." *Physical review letters* 112.14 (2014): 143903.

The dynamic equation becomes:

$$i\omega a_1 = (i\omega_1 - \Gamma_1 - \gamma_1)a_1 + i\sqrt{\gamma_1}S_{in} + i(k + ie^{-i\phi}\sqrt{\gamma_1\gamma_2})a_2$$
$$i\omega a_2 = (i\omega_2 - \Gamma_2 - \gamma_2)a_2 + i(k + ie^{-i\phi}\sqrt{\gamma_1\gamma_2})a_1$$

The solution of the amplitude:

$$a_1 = \frac{-i\sqrt{\gamma_1}S_{in}}{(i(\omega_1 - \omega) - \Gamma_1 - \gamma_1) + \frac{(k + i\sqrt{\gamma_1\gamma_2})^2}{(i(\omega_2 - \omega) - \Gamma_2 - \gamma_2)}}$$
$$a_2 = \frac{-\sqrt{\gamma_1}S_{in}(k + i\sqrt{\gamma_1\gamma_2})}{(i(\omega_1 - \omega) - \Gamma_1 - \gamma_1)(i(\omega_2 - \omega) - \Gamma_2 - \gamma_2) + (k + i\sqrt{\gamma_1\gamma_2})^2}$$

$$a_1 = \frac{\text{dirving}}{\text{bright atom} + \frac{\text{coupling}}{\text{dark atom}}}$$

$$a_2 = \frac{\text{dirving}}{(\text{bright atom})(\text{dark atom}) + \text{coupling}}$$

The reflection and transmission relation

$$r = \frac{i\left(\sqrt{\gamma_1}\, a_1\right)}{S_{in}}$$

$$t = e^{-i\phi} + \frac{i\left(\sqrt{\gamma_1}\, e^{-i\phi} a_1\right)}{S_{in}}$$

(1 driving term)

$$r = \frac{i\left(\sqrt{\gamma_1}\, a_1 + \sqrt{\gamma_2} e^{-i\phi} a_2\right)}{S_{in}}$$

$$t = e^{-i\phi} + \frac{i\left(\sqrt{\gamma_1}\, e^{-i\phi} a_1 + \sqrt{\gamma_2} a_2\right)}{S_{in}}$$

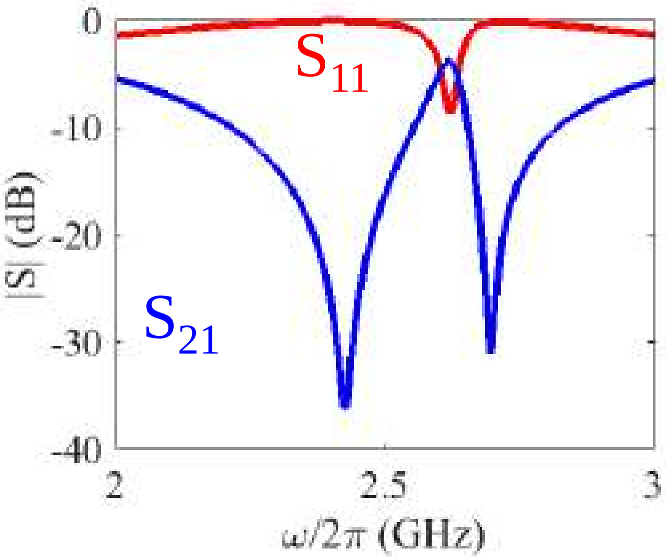
(2 driving term)

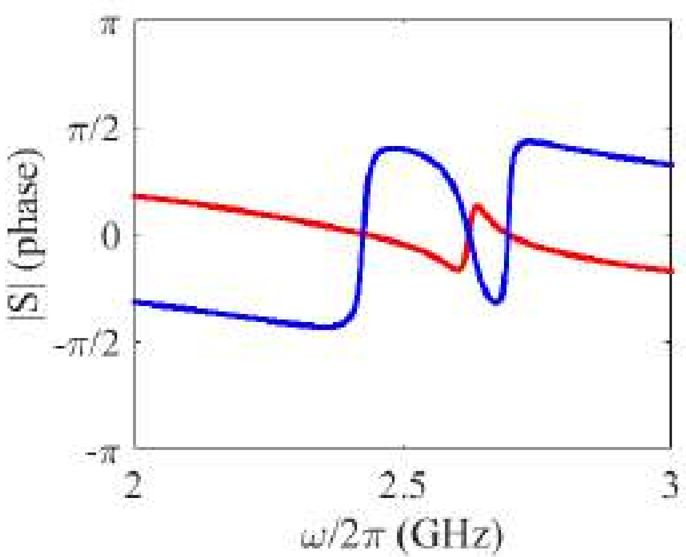
$$S_{11} = \frac{\gamma_1}{\left(i(\omega - \omega_1) + \Gamma_1 + \gamma_1\right) + \dfrac{\left(\kappa + i\sqrt{\gamma_1\gamma_2}\, e^{i\phi}\right)^2}{i(\omega - \omega_2) + \Gamma_2 + \gamma_2}}$$

$$S_{21} = 1 - \frac{\gamma_1}{\left(i(\omega - \omega_1) + \Gamma_1 + \gamma_1\right) + \dfrac{\left(\kappa + i\sqrt{\gamma_1\gamma_2}\, e^{i\phi}\right)^2}{i(\omega - \omega_2) + \Gamma_2 + \gamma_2}}$$

$e^{-i\phi} = i$ Coherent / near field coupling

$e^{-i\phi} = 1$ Dissipative / far field coupling





C: capacitors on Hyperbolic Metamaterials (HMM)
This changes the k in HMM, therefore $e^{i\phi}$

Field distribution measurement setup

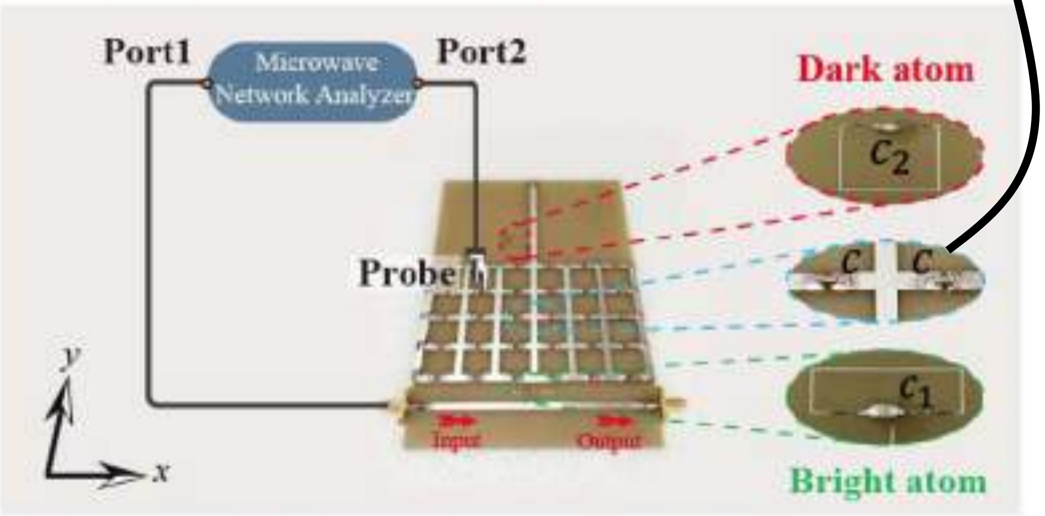
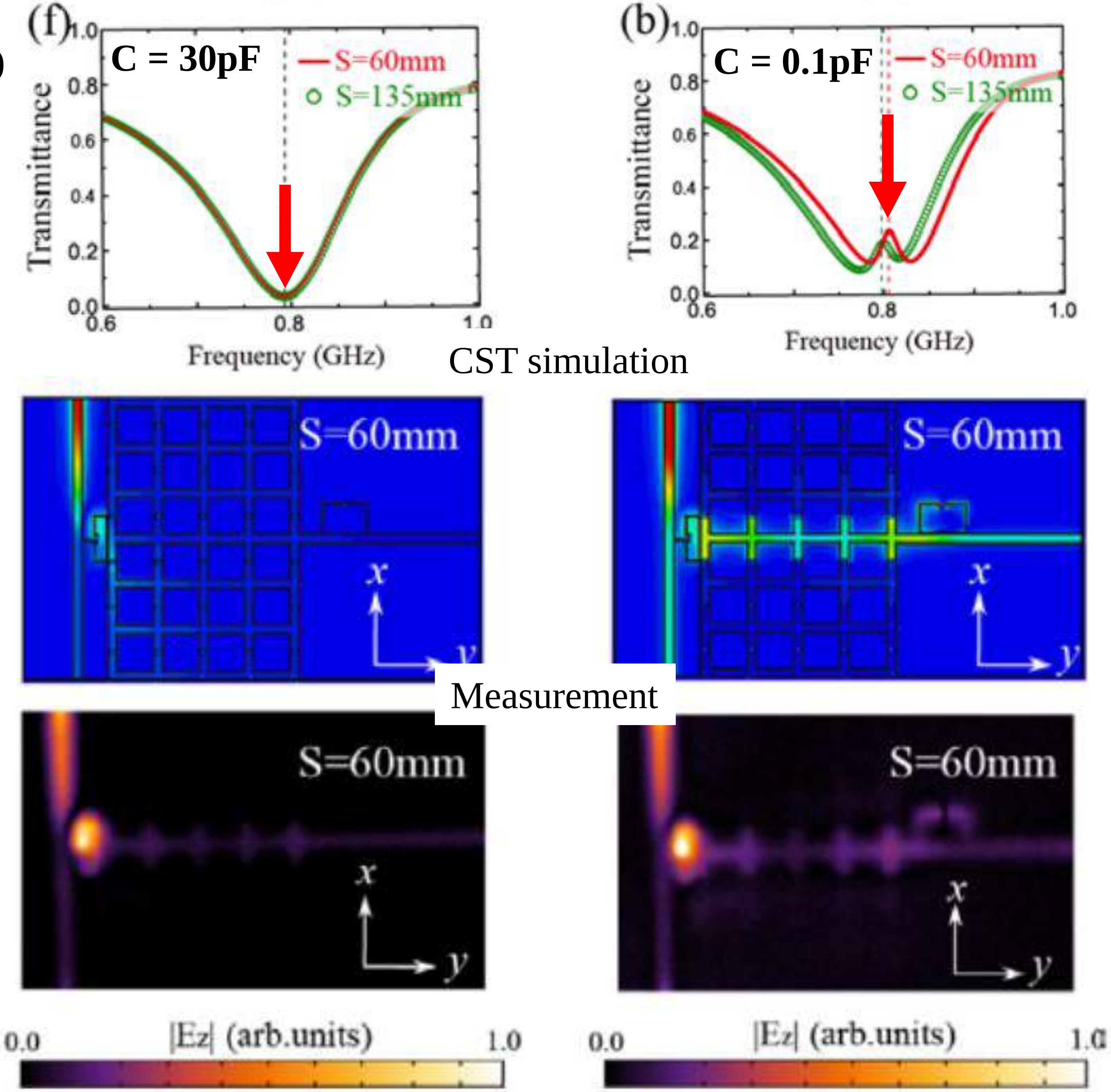


Fig. 4. Schematic of the structure to realize a long range EIT. Bright atom, dark atom and the unit of HMM are enlarged in the left, respectively.

Long distance EIT has been achieved



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Manipulating electromagnetic responses of metal wires at the deep subwavelength scale via both near- and far-field couplings

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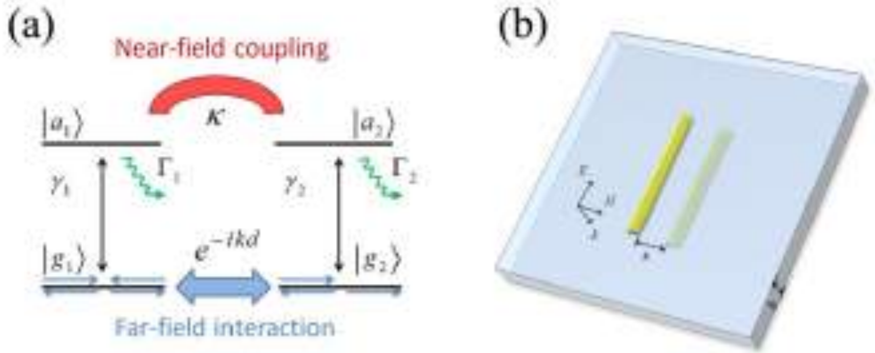


ously in this region: on one hand, the overlapping of strong and localized evanescent fields leads to near-field coupling;

also interacts with another resonator. The near-field coupling leading to energy level splitting has been well explained by

on the other hand, the resonator couples to the external electromagnetic fields and re-radiates propagating wave, which also interacts with another resonator. The near-field coupling

analogy with the molecular orbital diagram.[25,26] The far-field coupling alters the linewidth of radiation and also varies the splitting. Specific cases with various d are shown in

Dynamic equations

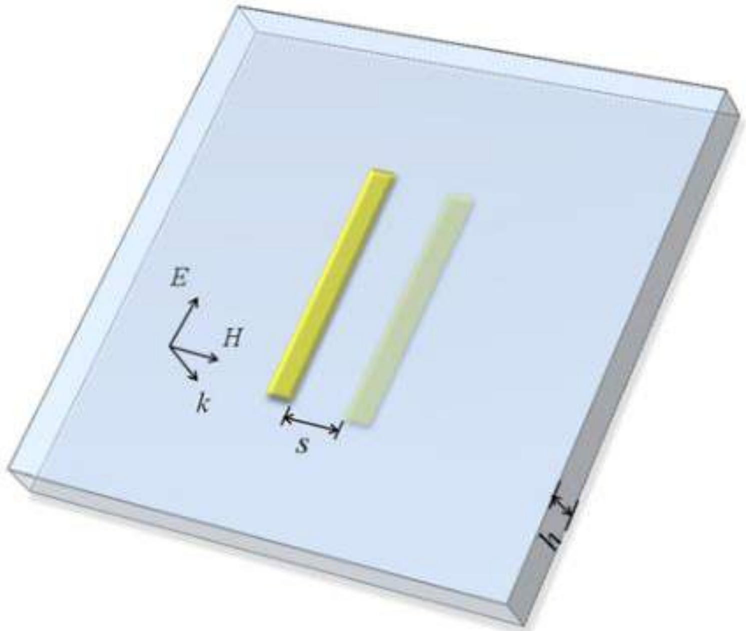
$$\frac{d\tilde{a}_1}{dt} = \overset{\omega_1}{\boxed{(i\omega_1 - \Gamma_1 - \gamma_1)}}\,\tilde{a}_1 + i\sqrt{\gamma_1}\tilde{S}_+ + i\overset{g}{\boxed{\left(\kappa + i\sqrt{\gamma_1\gamma_2}e^{-i\phi}\right)}}\,\tilde{a}_2$$

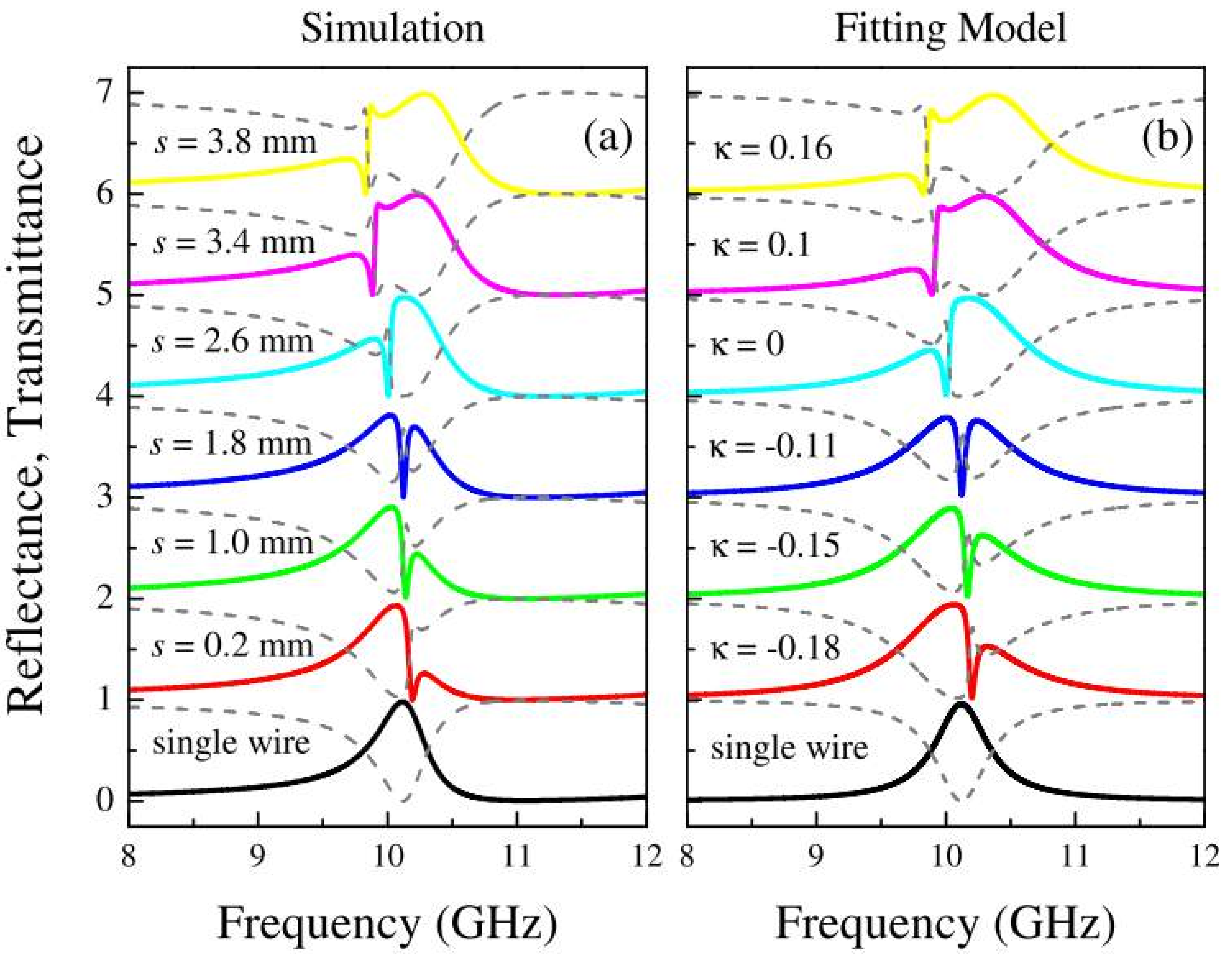
$$\frac{d\tilde{a}_2}{dt} = \underset{\omega_2}{\boxed{(i\omega_2 - \Gamma_2 - \gamma_2)}}\,\tilde{a}_2 + \boxed{i\sqrt{\gamma_2}\tilde{S}_+} + i\left(\kappa + i\sqrt{\gamma_1\gamma_2}e^{-i\phi}\right)\tilde{a}_1$$

Additional driving term

$$\tilde{a}_1 = \frac{ig\sqrt{\gamma_2}\tilde{S}_+}{\boxed{w_1}\boxed{w_2} + g^2} - \frac{\sqrt{\gamma_1}\tilde{S}_+}{\boxed{\omega_1} + \frac{g^2}{\boxed{w_2}}}$$

$$\tilde{a}_2 = \frac{ig\sqrt{\gamma_1}\tilde{S}_+}{\boxed{w_1}\boxed{w_2} + g^2} - \frac{\sqrt{\gamma_2}\tilde{S}_+}{\boxed{\omega_2} + \frac{g^2}{\boxed{w_1}}}$$

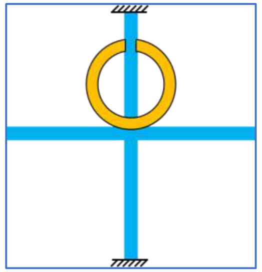




Why the sign of κ change

more flexible ways of tailoring lineshapes. We note that the cause of the sign change of κ is different from that in Refs. 25 and 26 which is the result of modified Coulomb interactions. Here, it attributes to the competition between intra-cell and inter-cell interactions. Numerical simulations are performed with a finite-integration-technique based EM solver (CST Microwave Studio). As shown in Fig. 3(a), the reflection spectrum for the sample with $s = 3.8$ mm has the similar lineshape with the model exhibited in Fig. 2. As s decreases to 1.8 mm, the splitting frequencies of the superradiant and subradiant modes get closer. With a further decrease of s, the superradiant mode moves to lower frequency while the subradiant mode moves to higher frequency, accompanied by a sign change of asymmetry parameter q of the Fano resonance.[27,28] Numerical fittings using Eqs. (1)–(4) are shown in Fig. 3(b). The fitted parameters (with the unit of GHz) are

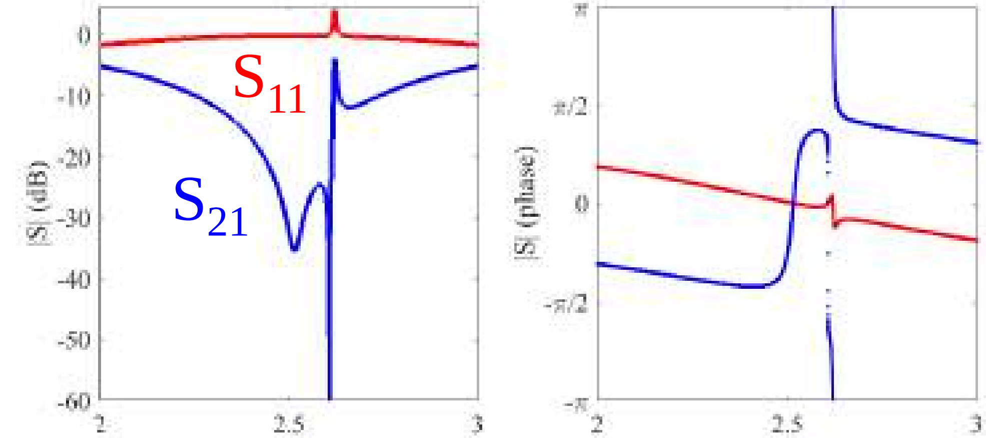
Problem of this model
: Reflection



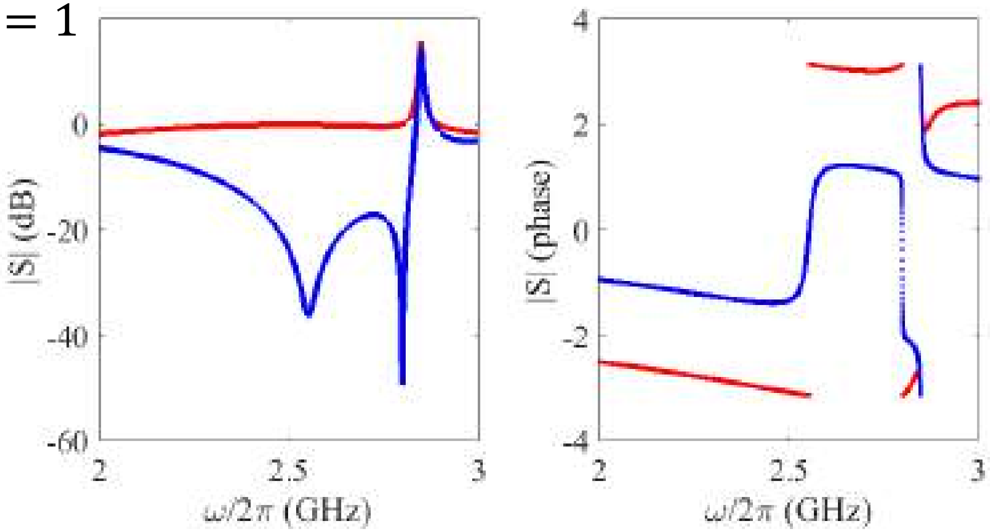
Experiment &
Simulation setup

Equivalent to YIPU's model

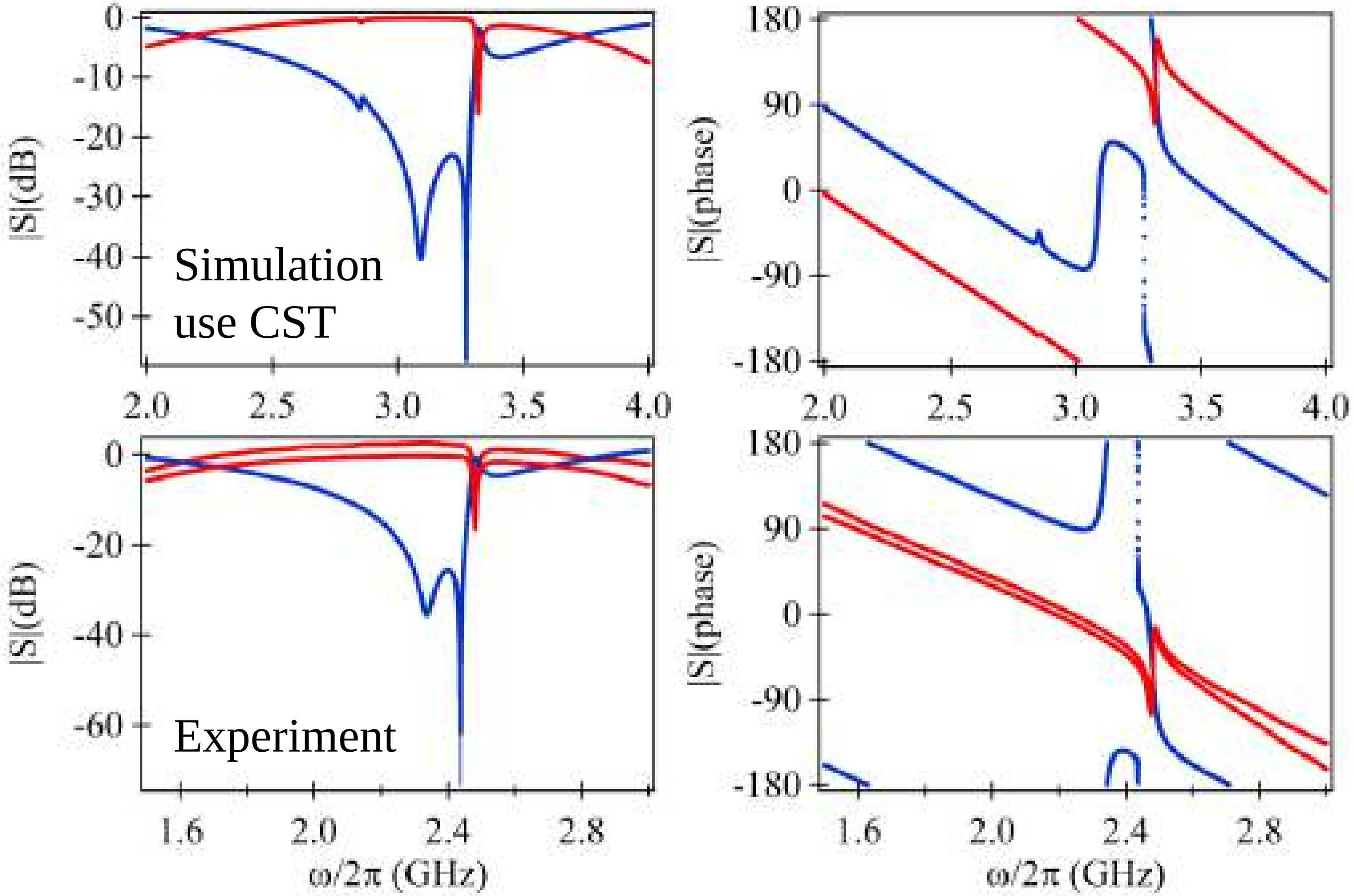
$e^{i\phi} = 1$ Calculated Agarwal's Model (1 driving term)



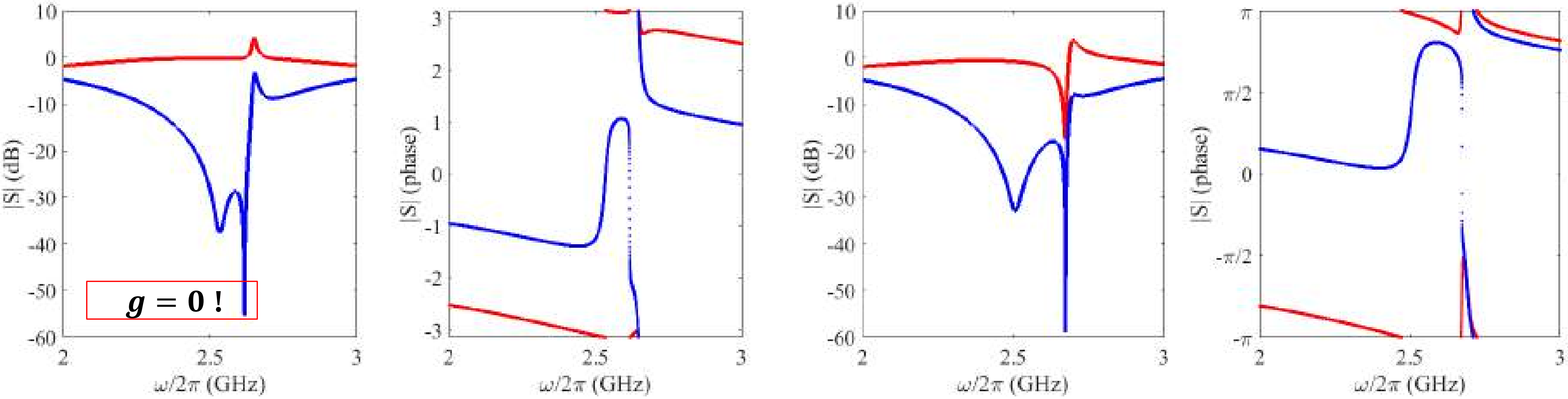
$e^{i\phi} = 1$



Calculated Chen Hong's Model (2 driving terms)



Calculated Chen Hong's Model (2 driving term without coupling) $i\phi = 1.545\pi$



Conclusion

1. Line shape of the level attraction can be explained in two different point of views.
    - Dissipative coupling (1 driving term)
    - Superposition of two Lorentzian resonance (2 driving term)
2. Currently, these models have difficulties explaining the reflection of the system.

Parameter I used for calculations

$$\gamma_1 \approx 700\ MHz$$
$$\gamma_2 \approx 14\ MHz$$
$$\Gamma_1 \approx 10.2\ MHz$$
$$\Gamma_2 \approx 2.5\ \ MHz$$
$$\kappa\ (J\ in\ YIPU's\ model\ ) = 14\ \mathrm{MHz}$$
$$\omega_1 \approx 2.5\ GHz$$
$$\omega_2 \approx 2.6 - 2.8\ GHz$$